

# True concurrency - from C.A. Petri to Telecom and Systems Biology

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September 18, 2014

- 1 The Token Game
- 2 Physics
- 3 Techniques and Engineering
  - Invariants
  - Unfolding
- 4 Coming to Life
- 5 Conclusion

# Nets and Concurrency

1 The Token Game

2 Physics

3 Techniques and Engineering

4 Coming to Life

5 Conclusion

# Carl Adam Petri, 12 July 1926 – 2 July 2010



## K o m m u n i k a t i o n . m i t A u t o m a t e n

Von der Fakultät für Mathematik und Physik  
der Technischen Hochschule Darmstadt

zur Erlangung des Grades eines  
Doktors der Naturwissenschaften  
{Dr. rer.nat.}

genehmigte  
Dissertation

vorgelegt von  
C a r l   A d a m   P e t r i  
aus Leipzig

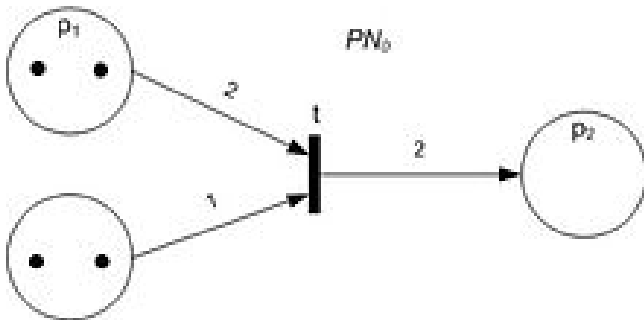
Referent: Prof.Dr.rer.techn.A.Walther  
Korreferent: Prof.Dr.Ing.H.Unger

Tag der Einreichung: 27.7.1961  
Tag der mündlichen Prüfung: 20.6.1962

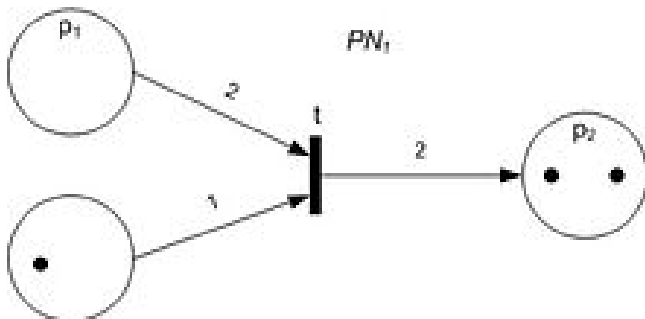
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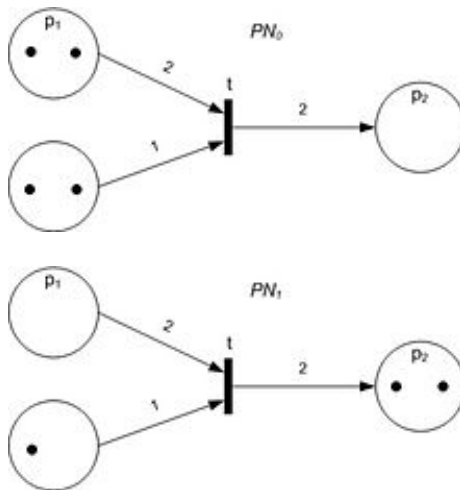
# The Token Game



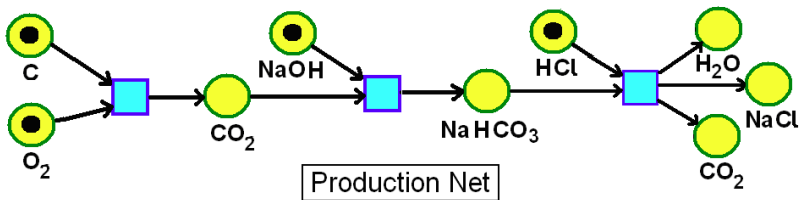
# The Token Game



# The Token Game



# First Intended Use



# Nets and Concurrency

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2 Physics

3 Techniques and Engineering

4 Coming to Life

5 Conclusion

## Minkowski-Diagrams ([Hermann Minkowski](#) 1908)

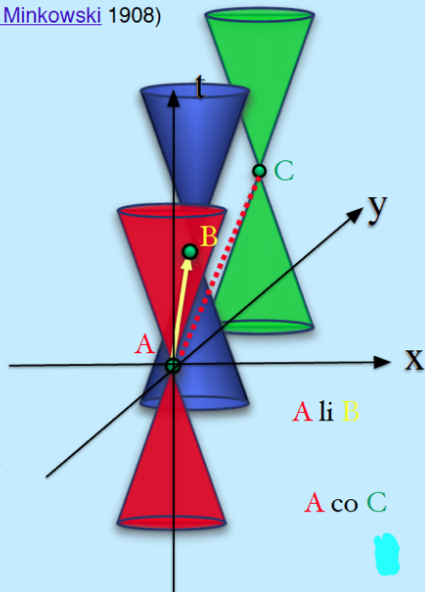
...,99,100,101,...



...,99,100,101,...



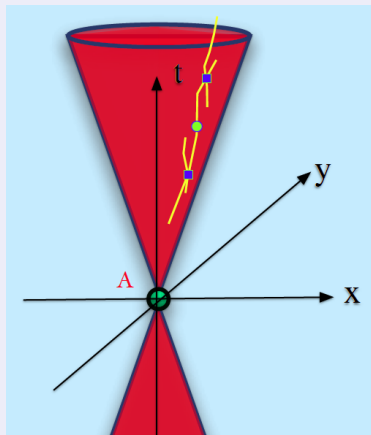
...,100,101,102,...



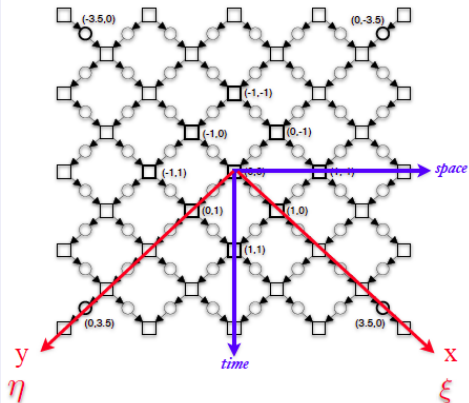
(Source: Petri and Valk 2008)

# From Space-Time to Nets

## Minkowski



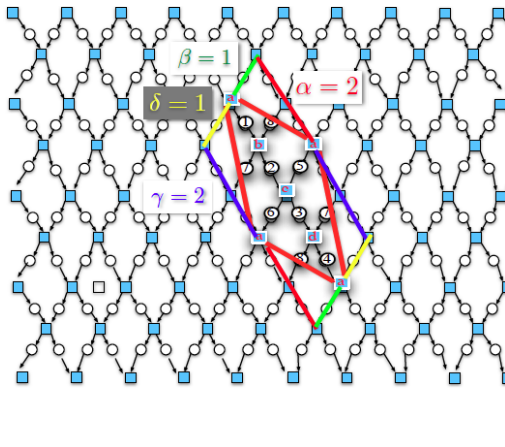
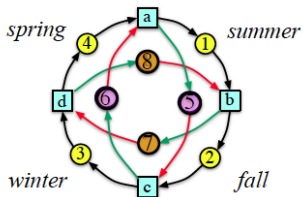
## Petri Grid



(Source: Petri/Valk 2008 and Valk 2013)

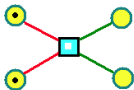
# From Big to Small

(2,1,2,1)



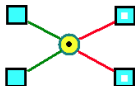
(Source: Petri/Valk 2008 and Valk 2013)

# From Geometry to Topology



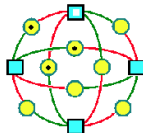
collection

GIVE

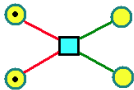


decision

TAKE

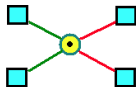


Oscillator



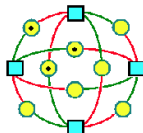
open subnet

The transition is  
completed by  
four states



closed subnet

The state is  
completed by  
four transitions

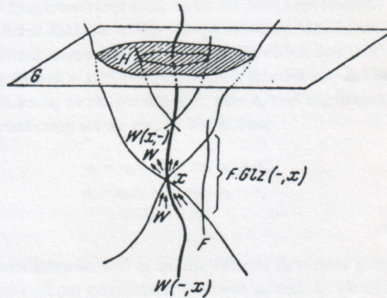


open subnet

The eight uncompleted  
states form the border

»Wir definieren: ein Raum ist eine Klasse von untereinander gleichzeitigen Weltpunkten, die mit jeder Weltlinie mindestens einen Weltpunkt gemein hat. ...

Hieraus folgt, daß jeder Raum mit jeder Weltlinie genau einen Punkt gemein hat«  
[Carnap-SL; 205]



**Abb. 1.4.2** Carnaps Originaldarstellung des topologischen Raumes  $G$  als einer Art Zeitscheibe. Es läßt sich leicht der Vorkegel, der Nachkegel und das Wirkungsgebiet von  $x$  erkennen. »Ein Raum ist sozusagen ein dreidimensionaler Querschnitt durch die vierdimensionale Raum-Zeit-Welt, und zwar quer zur Zeitrichtung, also so, daß er alle Weltlinien schneidet.« [Carnap-SL; 205]

# Physics and Concurrency

## Concurrency

- Relativistic Space-Time:
  - You don't see what is going on at Alpha Centaury **right now**
- Quantum systems:
  - Uncertainty prevents you from obtaining a **full global instantaneous snapshot**

## Some goals from Petri's program

- Axiomatic **relational** theory of concurrency (Carnap's spirit)
- Capture Lorentz transforms etc
- Build finitary but continuous mathematical picture of physics

## Here:

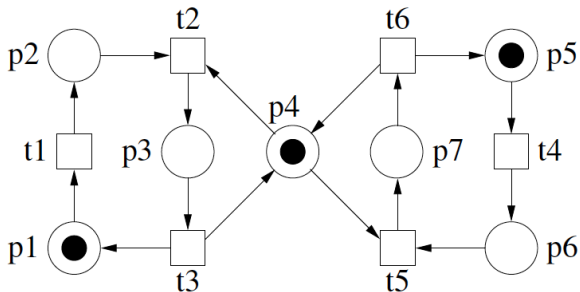
### PN Techniques and Concurrency in

- Engineering
- Life Sciences

# Nets and Concurrency

- 1 The Token Game
- 2 Physics
- 3 **Techniques and Engineering**
  - Invariants
  - Unfolding
- 4 Coming to Life
- 5 Conclusion

# Formalize !



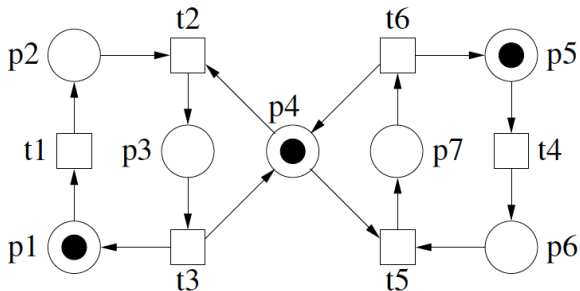
**NET :**  $N = (P, T, F)$

- $P \cap T = \emptyset$ , and
- $F \subseteq (P \times T \cup T \times P)$

**$\mathcal{N} = (N, M_0)$  is a Petri net iff**

- $N$  is a net
- $M_0 : P \rightarrow \mathbb{N}_0$  a marking of  $N$

# Matrix Representation



## Incidence Matrix

$$\begin{pmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & -1 & 10 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix} \quad 0$$

# Incidence Matrix and Invariants

Incidence Matrix  $N \in \mathbb{Z}^{|P| \times |t|}$

$$N_{(p,t)} = \begin{cases} -1 & : & p \text{ } F \text{ } t \\ 1 & : & t \text{ } F \text{ } p \\ 0 & : & o/w \end{cases}$$

(note: we assume there are no loops)

## State change equation

For  $X, M, M' \in \mathbb{Z}^{|P|}$ ,  $M \xrightarrow{t} M' \Leftrightarrow X^\top M' = X^\top M + X^\top W(t)$

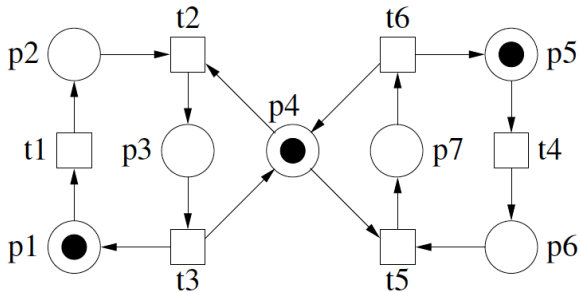
## Definition

A non-negative and non-zero solution of

$$X^\top W(t) = 0$$

is a **place invariant** of  $N$ .

# Running example



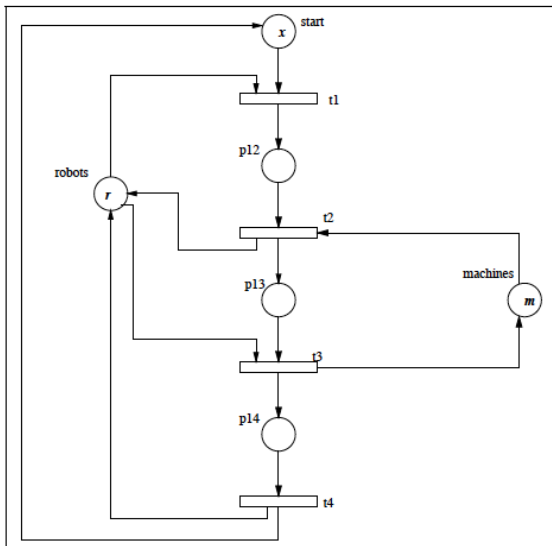
For all  $M \in R(N, M_0)$ ,

$$M(p_1) + M(p_2) + M(p_3) = 1$$

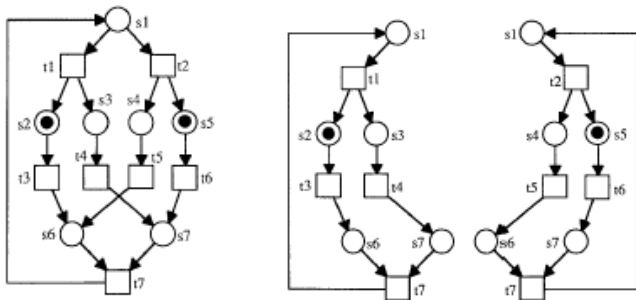
$$M(p_5) + M(p_6) + M(p_7) = 1$$

$$M(p_3) + M(p_4) + M(p_7) = 1$$

# Production Systems



# $T$ - Invariants



$J \in \mathbb{N}_0^{|T|}$  is a  $T$ -invariant of  $N$  iff

$$A^t \cdot J = 0$$

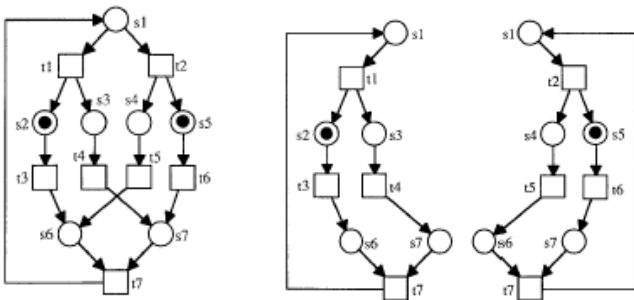
If  $M_0 \xrightarrow{\sigma_J}$  for some  $\sigma_J \in T^*$  such that

$$\forall t \in T : J(t) = |\sigma_J|_t,$$

then

$$A^t \cdot J = 0 \Rightarrow M_0 \xrightarrow{\sigma_J} M_0.$$

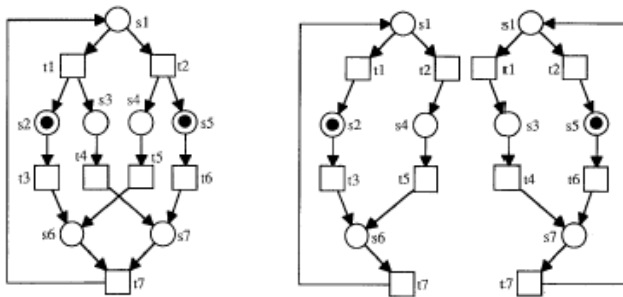
# Comparison of Invariants



## $T$ -invariants

- Help find state-reproducing behaviors
- Exhibit "typical" processes
- Give hints on **liveness**

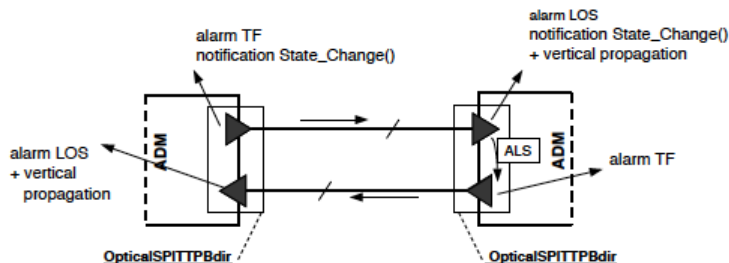
# Comparison of Invariants



## $P$ -invariants

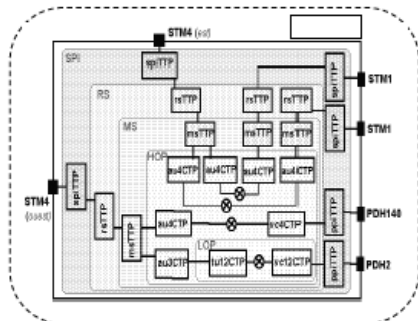
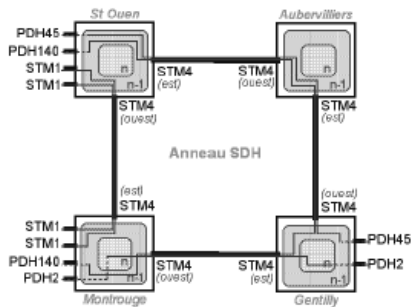
- Help express properties satisfied by **all** reachable markings
- Tell if the system is **bounded** → safety properties
- May allow decomposition
- Useful in control and supervision
- Next: Telecommunications , or: when one needs **unfoldings**

# Telecommunications : need Fault Diagnosis



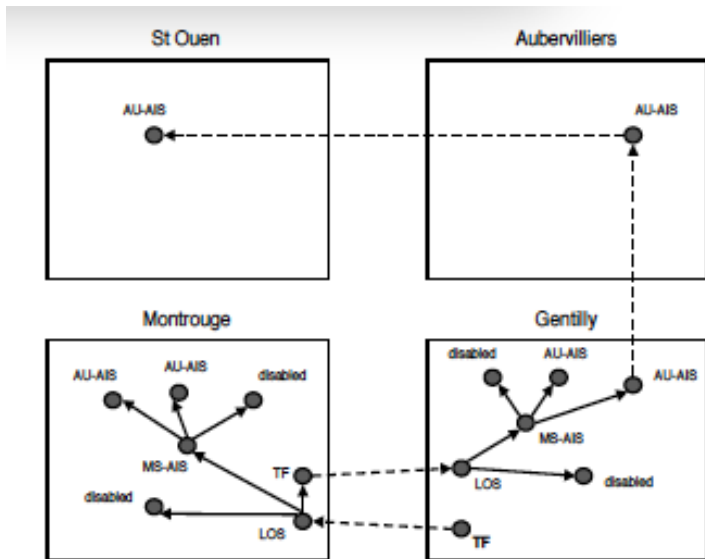
(Source: Benveniste et al, 2003)

# Seeking a Needle in a Haystack



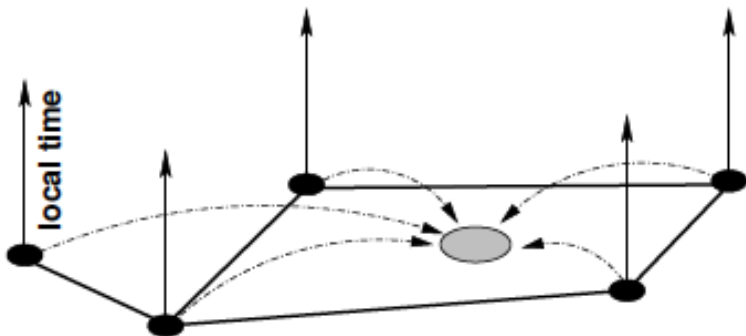
(Source: Benveniste et al, 2003)

# Seeking a Needle in a Haystack



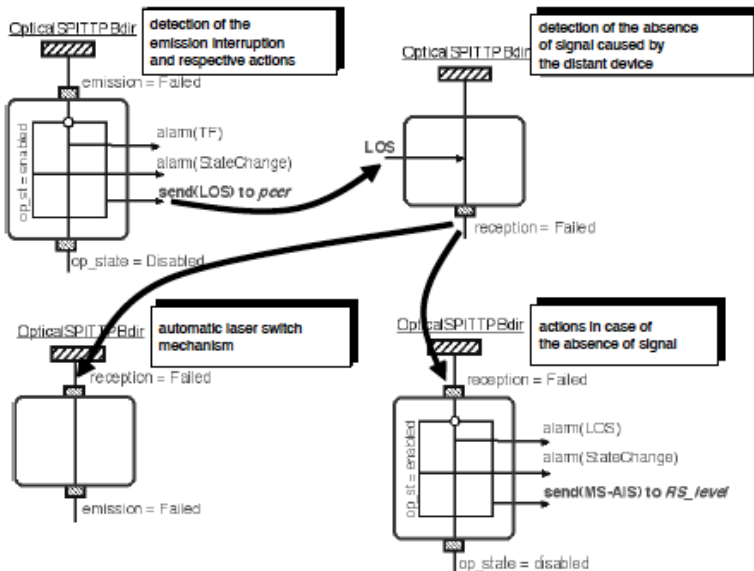
(Source: Benveniste et al, 2003)

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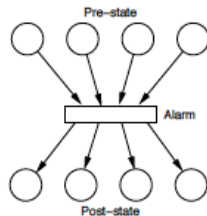
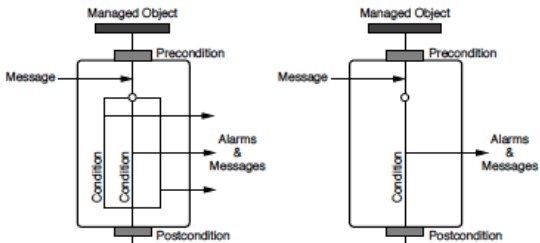
(Source: Benveniste et al, 2003)

# Reduce Complexity: Fault modeling



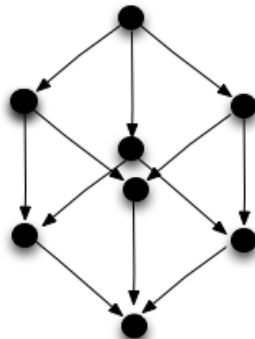
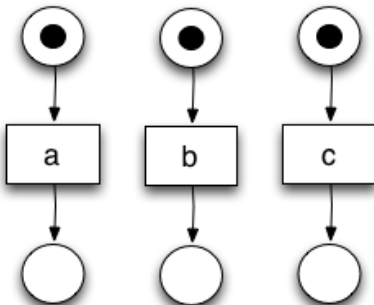
(Source: Benveniste et al, 2003)

# Reduce Complexity: Fault modeling



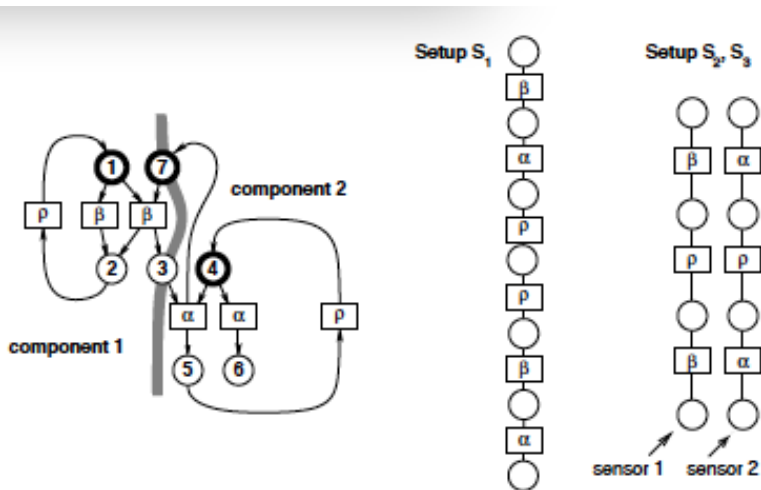
(Source: Benveniste et al, 2003)

## Reduce Size: True Concurrency



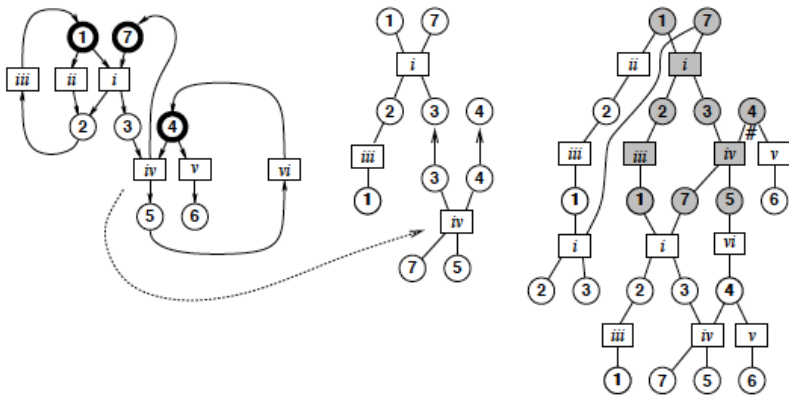
(Source: Benveniste et al, 2003)

# PN as Propagation + Observation Model



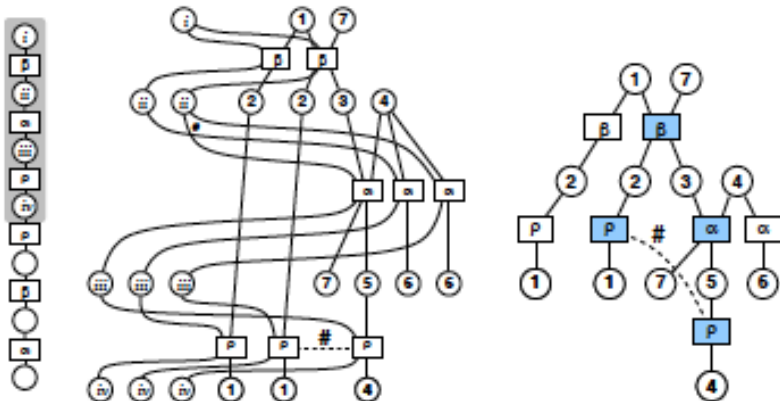
(Source: Benveniste et al, 2003)

# Finding behaviors: unfold



(Source: Benveniste et al, 2003)

# Finding explanations: Correlate and unfold



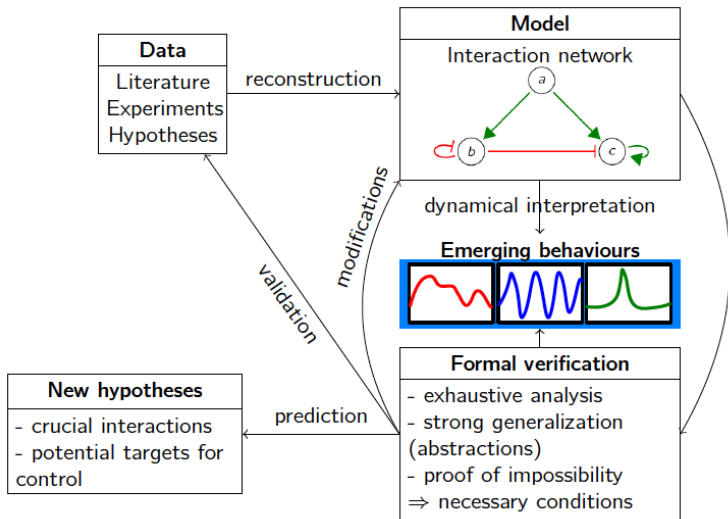
(Source: Benveniste et al, 2003)

# Nets and Concurrency

- 1 The Token Game
- 2 Physics
- 3 Techniques and Engineering
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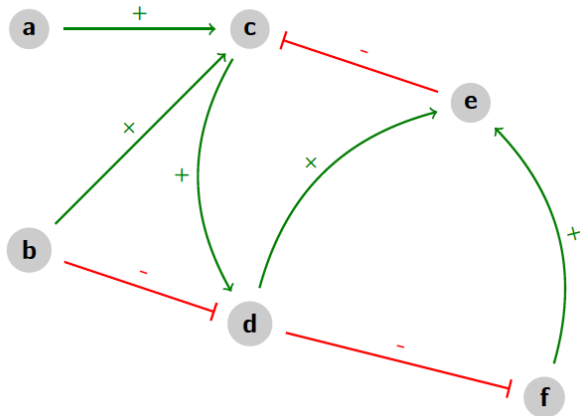
# Systems Biology needs Formal Methods

**Aim:** understand, analyse, control emerging dynamics.



## Biological Networks

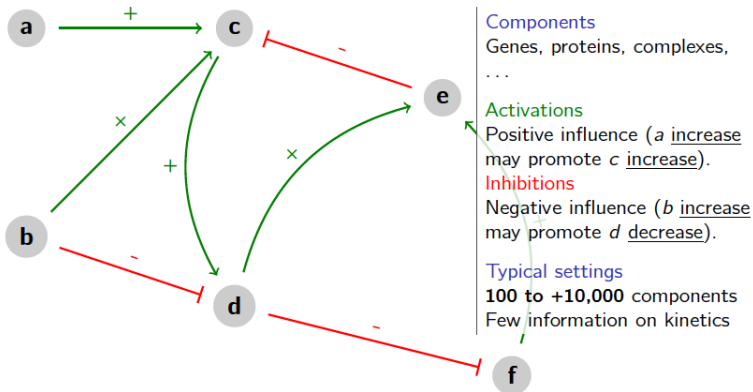
E.g., Gene Regulatory Networks, Signalling Networks



Source : L. Paulevé

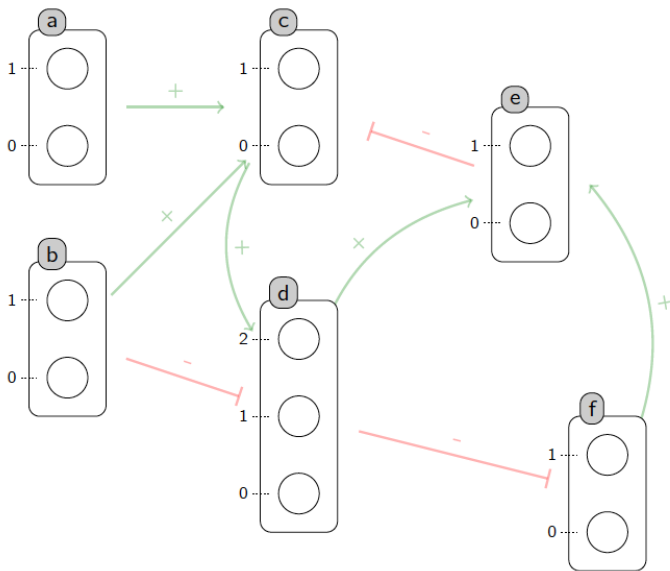
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E.g., Gene Regulatory Networks, Signalling Networks



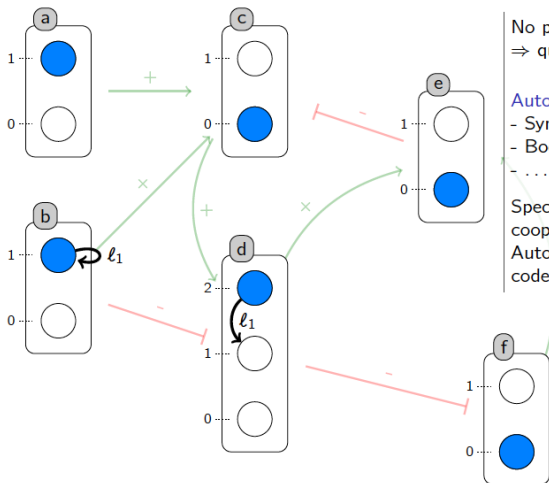
Source : L. Paulevé

# Qualitative Models for Biological Networks



Source : L. Paulevé

## Qualitative Models for Biological Networks



No population  
 $\Rightarrow$  qualitative levels

### Automata networks

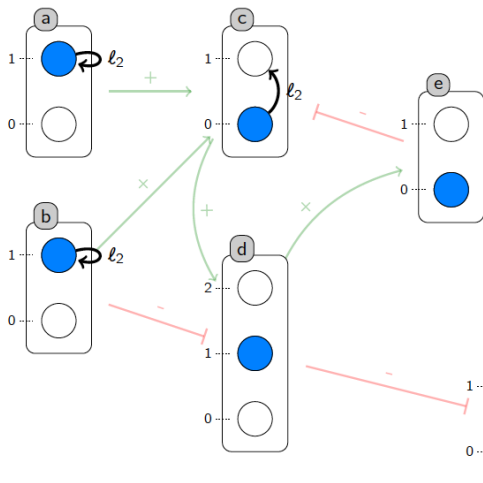
- Sync / async
- Boolean/multi-valued
- ...

Specify partial or complete cooperations.

Automata may be used to encode intermediate complexes.

Source : L. Paulevé

## Qualitative Models for Biological Networks



No population  
 $\Rightarrow$  qualitative levels

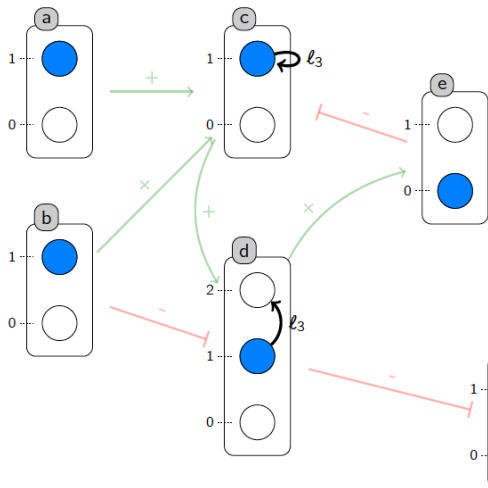
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Source : L. Paulevé

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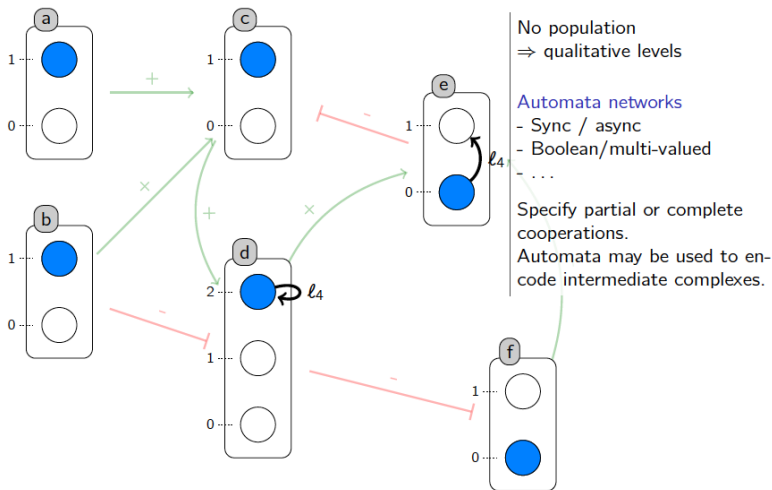
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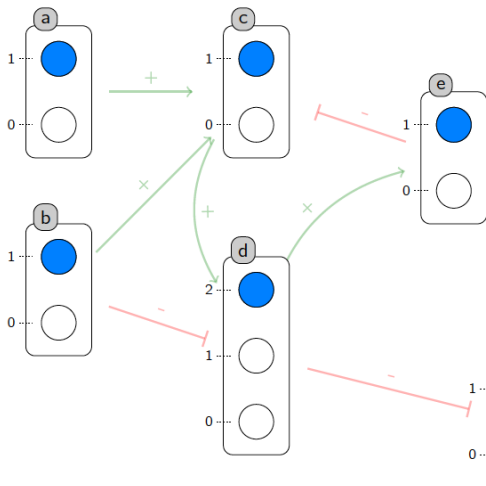
Source : L. Paulevé

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Source : L. Paulevé

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No population  
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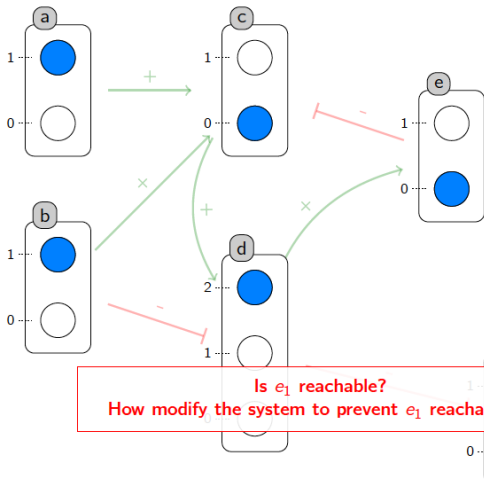
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Source : L. Paulevé

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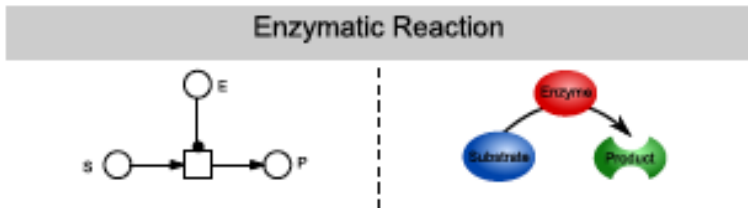
- Sync / async
- Boolean/multi-valued
- ...

Specify partial or complete cooperations.  
 Automata may be used to encode intermediate complexes.

Is  $e_1$  reachable?  
 How modify the system to prevent  $e_1$  reachability?

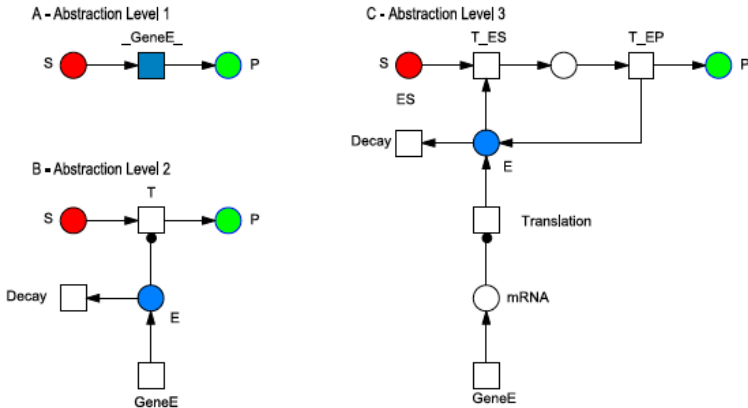
Source : L. Paulevé

# Petri Nets are a Natural Choice !



# Building complex networks

## Enzymatic Reaction Coupled with Gene Expression



# What For ?

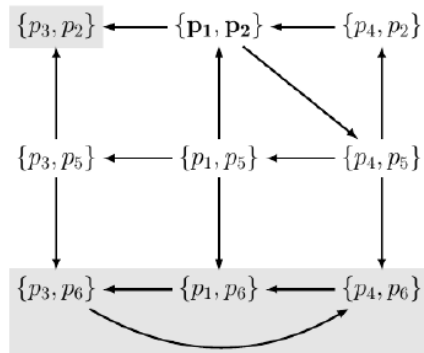
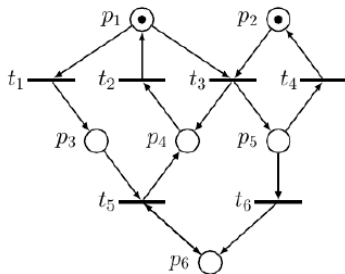
## Compute Attractors

- Terminal strongly connected components of State graph
- E.g.:
  - Stable state of a cell after de-differentiation
  - Reachable mutations

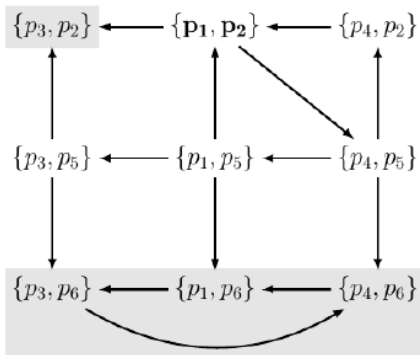
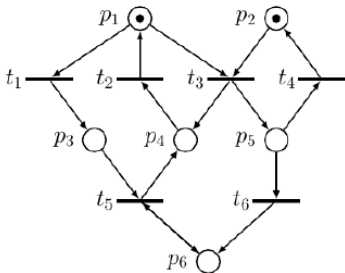
## Identify Cut Sets

- Sets of factors/actions/ ... that are necessary to reach some attractor
- E.g.:
  - Interventions to prevent a mutation
  - Medication to enforce a certain "healthy" attractor
  - Cell re-programming: move from one attractor to another

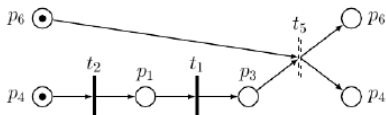
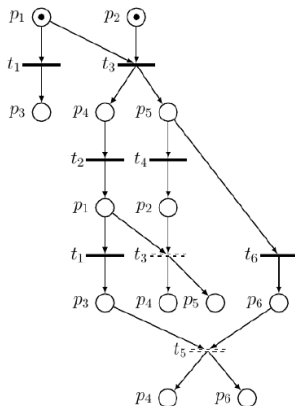
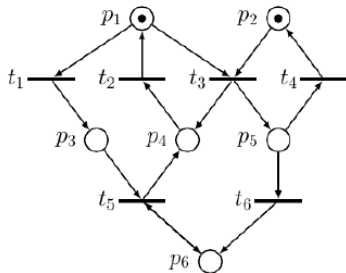
# The Petri View: State Graph ...



# The Petri View: State Graph ...



# ... vs Unfolding



# Some More Leads for PNs in Biology

## Unfoldings

- Accelerate search of attractors
- Find cut sets for control
- Understand cell reprogramming
- ...

## Compositionality, Invariants

- Synthetic genetic circuits
- Regulatory networks analysis
- Control
- Monitoring
- ...

# Nets and Concurrency

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# Conclusion: What was mentioned ...

PNs and concurrency are

meaningful in

- Physics
- Engineering
- (Chemistry)
- Biology

Concurrency

- is inherent
- is intuitive
- is helpful to save time

## ... and what wasn't mentioned

### More models and mathematics

- Contextual nets, time(d) nets, stochastic nets, fluid nets ...
- High-level nets, nested nets
- Max-Plus analysis

### More applications

- Verification of protocols, distributed algorithms, ...
- Workflows
- Railway Networks
- ... and everything I forgot

### References

... follow on next slides

### What Remains to be Said

BIG THANKS !!

# References I : Papers

On the subjects here:

- T. Murata. *Petri Nets: Properties, Analysis and Applications*. Proceedings of the IEEE, Vol. 77, No 4, April, 1989, pp. 541-580.
- J. L. Peterson. *Petri Net Theory and the Modeling of Systems*. Prentice-Hall, N.J., 1981, ISBN: 0-13-661983-5.
- W. Reisig. *Petri Nets, An Introduction*. EATCS, Monographs on Theoretical Computer Science, W.Brauer, G. Rozenberg, A. Salomaa (Eds.), Springer Verlag, Berlin, 1985.
- C.A. Petri. *Nets, time and space*. Theor. Computer Science Vol. 153, 199 pp 3-48
- E. Fabre, A. Benveniste, S. Haar and C. Jard. *Distributed monitoring of concurrent and asynchronous systems*. Discrete Event Dynamic Systems: Theory and Applications 15(1), pages 33-84, 2005.
- Th. Chatain, S. Haar, L. Jezequel, **L. Paulevé** and S. Schwoun. Characterization of Reachable Attractors Using Petri Net Unfoldings. In CMSB'14.

## References II : Online

- Online tutorial:  
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